

MARQUETTE

BUSINESS REVIEW

A JOURNAL OF FUNDAMENTAL BUSINESS PRINCIPLES

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SMALL BUSINESS: POLITICAL FRONT OR IDEOLOGICAL PROBLEM?

Edward Wisnewsky*

A report of the Select Committee on Small Business of the United States Senate says:

A few years hence historians of the American economy may mark down 1956 as the year in which small business for the first time became recognized as a major factor in our national life.¹

It is evident that in recent years more attention has been given to the fact that large business dominates our economy. There is considerable argument as to what this trend means, and what if anything ought to be done about the matter. Those economists following Professor Schumpeter essentially think nothing at all can be done about what they see as an evolutionary process, but there are others who disagree.

Those who disagree do so for a variety of reasons, but in this essay we shall be concerned only with the belief that changes in existing laws will be sufficient to halt the march towards eventual socialism. In particular, our concern is with the thesis that what is needed is a new doctrine of the proper composition of our economic structure for future legislative and judicial use, a thesis which implies that small business should be a "major factor" in our society.

This is not an academic matter, for a Select Committee of the United States Senate issued annual and thought-provoking reports on the condition of small business. The Small Business Administration, once a temporary agency, now has millions of dollars and permanent status as a branch of the executive department. Can all this be dismissed as "just more politics," as some critics and commentators claim?² Or is it to be construed as a genuine sign of growing anxiety over the nation's business system, an anxiety which will inevitably affect that system and centering, in Max Lerner's words, "about the ways in which it achieves stability."³

What, also, is to be made of the institutional advertising featuring the connection of big business with small business? Is it all a form of preventive brain-washing? If so, why?

The answers do not come easily. The controversy over the place of small business has many facets. It involves not only complex economic

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questions, but also involved sociological questions. We shall see that when debate centers about the proper role of small business in society, matters of basic significance to intellectuals as well as to businessmen are involved. On this ground alone some scrutiny of the "small business problem" would be justified.

This article attempts to shed light on various pertinent questions and to provide a neutral background for the general evaluation of small business goals of both political leaders and of businessmen. The background to be presented is admittedly more useful for observers than for those attempting to rally public support. At the same time, some effort is made to point out what champions of economic reform, in the name of small business, may need to consider more fully in the development of their programs.

To start, we need a definition of small business.

What is Small Business?

The term "small business" as a categorical notion for use in framing legislative and administrative policy provided a useful classification during World War II days for allocating defense orders so as to preserve small manufacturing plants. Its use established an additional precedent for public responsibility for preservation of the elements of the business economy. And it is because there are such precedents ranging from anti-trust laws on the national scale to anti-chain store legislation on the narrower territorial basis that legislative intent plays a large role in business society. Current legislation conferring authority and responsibility on the Small Business Administration must be viewed in the light of such public policy, even though, as is apparent from the comments of the Select Committee, there seems to be a difference between what the senators think it is and what the Small Business Administration practices.⁴

Yet in spite of precedents, recent Congressional efforts to broaden the base of the definition and create a meaningful concept for peacetime policy-making differentiating the parts of the business world have not been satisfactory. "Small" is not sufficiently precise a criterion to have methodological significance, although it may, by reasons of vagueness, be useful for other purposes. This difficulty in agreeing on what is meant by small business is in itself quite significant.

One of the causes of difficulty lies in the attempt to evolve a descriptive formula which includes the multitude of small retailers as well as the employers of 50 or more persons. It is quite apparent that a change in our distribution system dating back to some twenty-five years ago has altered the economic function of such small retailers. Today's distribution system is essentially buyer-oriented rather than seller-oriented, and this has diminished (if not destroyed) a traditional function prop for the economic position of the small retailer.

To use the words of David R. Craig and Werner K. Gabler our marketing system today is guided by large retailers or combinations of

retailers rather than by manufacturers or wholesalers.⁵ The result is that, competitively, the small independent retailer is out-classed and in many cases outmoded as a distribution element of importance.

We can simplify matters considerably by ignoring the small independent retailer whose marketing role depends upon the preservation of traditional wholesaler functions. Congressional leaders, however, cannot take such a step publicly no matter if it does serve to sharpen and clarify the central issues. While Congress continues to have difficulty in defining small business we may be free to speculate independently in this concept area, anticipating the root of the anxiety about the role of small business in the process, even as we exclude the special problems of the small retailers.

In my opinion, based on observation as a working member of the small-business community, what is often meant by "small business" is that segment of business enterprise which is closely or privately held rather than widely or publicly owned, and further, which is so because of necessity rather than choice.

The qualification is important because it draws attention to the possibility that not every small business is to be regarded as seed out of which a larger business might develop. This seed-notion is prevalent in community industrial development programs. While it might prove important as a premise in the final defense of any small business program, it confuses a discussion of fundamentals.

The Effects of Concentration on Ownership Nature

The use of an idea which divides the business community on the basis of the nature of the ownership involved has several immediate advantages.

The emphasis on the nature of ownership involved in business provides one clue as to the reasons for the sentimental regard in which small businessmen are often held. It is this regard which leads political candidates to proclaim the virtues of the industrious workingman, the deserving farmer, the struggling businessman and to class all three together in a vague way as economic underdogs.

The traditional family owned enterprise could be widely understood. It was limited in scope and motivated in an easily comprehended way. In today's world such enterprises appear to be static and unprogressive but they are still found as important economic factors in French Canada and Puerto Rico, for example, and they were important factors not too long ago in our own economy. Private ownership of this type has its roots in the era when political liberalism was hammered out. Further, it is linked to what might be called Utopian capitalism in which almost every man could become a capitalist by practicing Franklin's virtues of thrift and hard work.

A second advantage of a definition based on type of ownership is that it connects us to the economic history of the immediate past. There

is no doubt that the main issues in the public debate over domestic political economy over the last half-century revolve about the differences in meaning to be linked to the changes in the function of ownership. There exists a considerable literature on the topic of the difference between the formal legal structure of business organizations and their functional nature.

This literature has made it evident that changes in the nature of ownership of business entities have profound effects both from the standpoint of economic performance and from the standpoint of social results. Thus, the definition advanced allows us to consider these differences in our discussion of the role of small business in society.

If we use privately owned business as a term to split our economic language map into two concept areas, we get away from the antagonistic portrayal of "small" versus "big", and its confusion-creating by-products. Size enters the picture only as a consequence of the type of ownership involved. In a rough way, then, we can differentiate between the two kinds of ownership according to these general traits:

In publicly owned enterprise, ownership claims are mainly satisfied with dividend payments, and their value lies in the capital gains possibilities of shares of stock. Ownership here is limited in role and formalized. In many ways in business practices, we find that such a recognition is present. The accounting practices which measure performance as a return on investment reflect public ownership effects.

In the privately owned enterprise there are no such formal limitations on the claims of ownership and its role in the conduct of the enterprise. It is even doubtful that the same accounting standards of success ought to be applied to such businesses.

Differentiating between types of ownership would permit further inquiry into the practices of large corporations but from a different orientation. In particular it would be useful to determine if some of the practices of our large corporations are due to size alone, or whether they are the results of the type of ownership involved. The main question, from the standpoint of prevailing legal and economic theory, becomes whether the concepts appropriate to the rational framework of formalized ownership inherently operate to reduce the regulative force of the competitive market -- a force which, theoretically, is the key dynamic element in a free-enterprise marketing system.

The acceptance of the definition presented here also serves to put the self-employed in still another economic category and contributes, thereby, further clarity to economic and social thinking and perhaps even to political activity as a result.

The real utility, however, of the definition proposed can only be demonstrated if it throws light on three important ways in which

business organizations can differ from the standpoint of the general society.

1. They can differ according to their size.
2. They can differ according to their life-span.
3. They can differ according to the social benefits they provide their members and the society at large.⁶

It is the first and third of these differences which will be mainly discussed in the following sections. It is not that the second is less important, for many of our current business and management problems seem to arise from the effort carried on in organizations seeking to perpetuate themselves. It is simply that this difference is of less significance for those issues mainly involved in "small business discussion."

Ownership and Size

From the standpoint of those concerned with the effect of "bigness," the prime difference between a privately owned business entity and a publicly owned enterprise is the presence of an effective limitation on the size of a privately owned enterprise. This limitation exists because of a generally accepted public policy holding that private fortunes must be limited in size through taxation.

While it can be argued that historically the anti-trust laws came first as factors expressing a public policy which sees danger in size, and the inheritance and income taxes came later in time, there was a difference in the intention of these means of economic control. The anti-trust laws were intended to limit business size; it is doubtful that this was the primary intent of the inheritance and income taxes although they obviously now have such an effect. It seems evident that the taxation measures now serve to limit size of privately owned enterprises more severely than the anti-trust acts limit the size of publicly owned entities, even though this may not be intentional.

The basic arguments for limiting business size because of the danger of monopoly are well known and need no repetition. They were developed in the period before 1900. But to this arsenal of arguments the past quarter century has added one more as the result of the economic collapse of 1929-1932.

Some critics of our economic order have argued that economic concentration in the 1920's "brought with it a short-sighted and muscle-bound private control over the economy that produced a rigidity in the price system primarily responsible for the extreme severity and depth of the 1929-1932 depression."⁷

Public Ownership and Social Effect

While the social effect of an economic order which places emphasis on wealth has been subjected to criticism with particularly biting attacks by Thorstein Veblen, something new has been added in our day. What

has drawn recent criticism is the effect of the organization of the large sized business enterprise on the white-collar worker and his supervisor.

No one pretends that this kind of criticism is based on economic theory, as was the attack on the relations between the corporation and its hourly-paid wage earner. The criticism owes more to Freud than it does to Marx or to idealistic socialism. What makes this new criticism important is that a doctrine of the social responsibility of managers, particularly applicable to managers of publicly-owned corporations, has been gradually taking shape.

There are many factors involved in this development. Not the least of these is that publicly owned corporations tend to operate as if they were institutional forms bent on the perpetuation of their existence. But further discussion would take us far afield; all that is required for our purpose is to recognize that as a result of this doctrine of social welfare there is a disposition to regard the manager of a large enterprise as a kind of quasi-trustee with at least a moral obligation to manage in a way accountable to the public at large.

This obligation has been expressed in different ways by different persons, for social responsibility is subject to a variety of specific interpretations, ranging from expression of guilt to a high level of altruism. Ordway Tead, a prolific writer on administration, puts it in this form:

... those who administer must assume a new solicitude as to how individual personalities can become enriched as they are necessarily immersed in corporate affairs. ⁸

It is significant that some managers of big business do, in fact, take the position that they have a quasi-public function and speak of the obligations of "business statesmanship." There is in much of this a tacit admission that there must be some correlation between the acts of big corporations, big unions and big government for the accomplishment of certain socially desired ends -- the enlargement of the working relationship between government and big business to include big labor is the new factor here.

What is implicit in the charge that big business has a social responsibility is a still vague appreciation that such organizations can accomplish certain things which apparently smaller organizations cannot do! And it is this fact which is important for determination of the role of small business today. If it is indeed true that there are social gains which can only be provided in the framework of large and self-perpetuating organizations, then privately owned business functions under a severe handicap. Let us turn to consider two aspects of our economic society connected to this possibility.

Job Variety and Personal Debt

While there exists disagreement as to what the vital political concern of our day is, it is not over-enlarging the means of the market place

by which individuals can free themselves from the economic limitations of their fathers. The development of large-scale publicly owned enterprises has substantially eliminated that need by the very process which reduced the opportunity for such acts,

The growth of large-scale industrial and commercial complexes has these results of significance to the present topic:

It has greatly extended the range of occupational opportunities of a socially desirable type. It has permitted and encouraged the specialization of individual talent and provided an important social position as a reward. The social position has cosmopolitan traits rather than parish characteristics.

It provides men with a relatively stable salaried income level in a commercially and politically urbanized area along with the uses of goods and services paid for through instruments of nation wide credit.

It is not just size which has produced such effects. The result is due to the combination of size with a rational development of organizational means for the attainment of a self-perpetuating form.

In the search to determine the proper role for small business these social gains are not to be minimized. Nor is too much to be made of the fact that they were gained by means not in conformity with the pieties and social virtues identified with the relatively isolated life in small towns and cities of a generation ago.

Each of these developments has a consequence of special importance for the small businessman.

Property rights are not as functionally important to the individual in metropolitan and industrialized sectors as they may still be in small population centers. Property is one means of attainment of desires, but not the most significant.⁹

Job stability and job description are more important for individuals than are property rights. The "fringe benefits," too, are now a part of the total package to be weighed and considered.

In our history, economic class stratification is at most temporary and has a limited political effect -- regarded more as the consequence of individual failure to take advantage of certain opportunities for development than as the result of oppression.¹⁰ The serious aspect of discrimination on the basis of minority traits is that it prevents the individual from reaping the social benefits of having taken advantage of existing avenues for self-development, avenues which are continually being enlarged. These social benefits are tied to the kind of a job a man can hold -- the kind of job opportunity an enterprise can offer. In general, such opportunities show a wider range in publicly owned entities.

This has greatly changed the value of formal education. We are

tending to accept the notion that motivation for learning in school is more significant for personal success than the on-the-job motivation of the past. It is simply impossible in many organizations to move into higher levels without certified evidence of formal educational training of a definite type. This condition is most likely to hold in publicly owned enterprise.

The other contribution, often ignored but quite important, made by large scale publicly owned organizations toward the improvement of the standard of living is the development of the consumer credit system. This is a major economic invention not at all adequately comprehended in our economic philosophy. This credit system has permitted men to use things they do not own in the traditional sense. It has even permitted recreation, travel and education to be financed on limited immediate cash resources. It connects the world of work to the world of leisure.

Installment buying makes job stability and promotion channels leading to higher incomes more important for most people than property rights. Agrarian bias about the intrinsic value of job stability colors judgment about participating in organizations where advantages are seemingly bought at the price of limiting the area in which individualistic preferences can operate. These advantages may be of greater importance in metropolitan society than the preservation of those agrarian freedoms which simply cannot be utilized effectively by the resident of a metropolis.

All of the above is intimately linked with the fact that publicly owned self perpetuating organizations have an inherent advantage over privately owned enterprises. That advantage lies in the fact that such organizations can obtain greater amounts of money to work with over longer periods of time. This is apparent in the oft heard complaint that small business suffers from the lack of adequate financing. But it is difficult to see how financing based on fifty or one hundred year time spans can be provided to privately owned enterprises on any rational grounds.

In summary, we can say:

The society of our giant business enterprises not only permits people to use property that they do not own but has created new professions insofar as community status is concerned. The property owner is eclipsed in importance in the local society by the paid executives from the ranks of middle management, provided that such middle management is in a national or international organization.

Parents, in directing and influencing the education of their children, have not missed this important fact, for it is the main road to economic and social stability today. There is no evidence that any other road promises as much gain of this type for as many persons.

Is There A Public Preference for Business Form ?

It is not only the functional differences between privately owned enterprises and publicly owned entities which must be considered in

arriving at some determination of the social role of small business; we must also consider certain aspects of the general climate of opinion which appear to favor one form over the other. These can be stated briefly:

The rationality of a creed which saw the world as a stage on which men were born to compete on the basis of thrift, self-interest, and initiative has been discredited. The prevailing rational temper favors association and cooperative action in the social sphere.

Organizational practices indicate reliance upon motivations and attitudes more complex and varied than Adam Smith's simple mechanism of self-interest.

It is plain that we are educating men to perform jobs which have no meaning except as organizational components. There is, therefore, in a great deal of our training the inculcation of ideas as to what an organization "ought to be like" along with techniques to be used by the person trained. This education gives its possessors a vested interest in certain kinds of organizational forms. Those forms appear to be more easily realized in the framework of a publicly owned enterprise than in a privately owned enterprise.

There is a growing public feeling that the real opportunities for prestige and for the attainment of desired personal expression all appear to lie in directions other than engagement in individual enterprise -- and this appears to be especially so for those persons with the highest educational levels.

What is particularly interesting here is that sons of businessmen are apparently not imitating the parental example, especially when given the benefit of college training. Students of organization structure implicitly accept the premise that for most men personality growth demands quite complex organizational connections. It is not only the "beat generation" which realizes the meaning of this, but it is the first generation for whom this is generally true even in adolescence.

The privately owned business area must be regarded as the last stronghold of the agrarian ideal of economic and political individualism generally prevailing in the period before the Civil War. This ideal placed the greatest value on the traits of will and single-minded purpose aimed at providing the maximum private benefit through disciplined individualist action. The self-sufficient Thoreau represents the highest flower here. As such a stronghold, it is often a center of romantic intellectual rebellion against urban practices.

It may well be that the real social importance of preserving small business opportunity is that in this way persons with modest education have an avenue in which to move toward higher social status and income. But more facts as to the characteristics of persons entering the business community by starting their own enterprises are required to support such a conclusion.

What matters is that the prevailing temper places a premium on

those political programs which are directed toward enlarging the supply of desirable jobs and stability of employment, or increasing tax revenues. This social temper essentially favors the establishment and growth of publicly owned production and distribution institutions. But it does so reluctantly and with uneasiness as to ultimate results. The capital gains element in our income tax structure is only one example of such favorable and directive treatment.

We can conclude that while public behavior testifies to a preference for the stable, self-perpetuating business form, there is no economic philosophy which expresses this preference in a politically potent form. In fact, what we have is a public philosophy which seems to prefer the competitive private enterprise form, but a public behavior pattern which takes the opposite course.

In addition, certain other realities demand attention from the students of the social role of small business, especially from those dedicated to the task of improving the climate for such enterprise.

One Possible Future for Small Business

If we are to take seriously the notion that public policy should be explicitly framed so as to enlarge the economic role of small business, attention has to be given to the obvious fact that, by and large, small business enterprises are notorious for weak management. The lack of special education in operating principles and management theory presented no real handicap to success when economic individualism was a functional ideology and a seller's market existed. This appears not to be the case today, if the proliferation of programs to bring about specialized operating "know-how" to business operators is any indicator.

The multiplication of trade associations and the fantastic growth in management education programs can be taken as acceptance of the thesis that small business enterprises are to be efficient first of all. There is, however, no reliable information telling us whether the decrease of small business opportunity, or its diminishing role in the economy, is due primarily to the limited managerial resources of small business entities, or to other factors more amenable to legislation.¹¹

The motive for increasing exposure of business managers to the scientific experts is partly a concern about the high failure rate among newly established firms. In manufacturing, for example, about 50 per cent of business failures during 1958 involved firms with an age of five years or less. The major cause for such failure, as given in Dun and Bradstreet's The Failure Record Through 1958 is described as incompetence. Surely we would not be far wrong in translating this to mean ignorance or unwarranted optimism.

The presumption in favor of such a translation is heightened by the 1944 survey of the National Opinion Research Center for the Committee for Economic Development which revealed that about 25 per cent of owners of small companies came from the ranks of the non-salaried employees in manufacturing concerns.¹² It is not surprising that here

the belief is quite strong that in business it is still possible to get to the top by starting on the bottom rung of the ladder and just working hard.

This belief is involved in the ambivalent attitude toward wages. There is some confusion as to whether it is output that is being paid for or whether the employer is really buying work input possibilities by paying for time spent. As a result, we see a situation in which communication is sometimes thought of in terms of maintaining some existing output level and at times as a technique for increasing work input, thus getting more for the wages provided. It is not too much to say that communication in smaller enterprises is regarded more as a system of motivation than as a system of control and coordination.

To this observer, it appears as if the current emphasis on educating the small businessman in scientific principles of management parallels the earlier efforts to preserve the small farmer through education in scientific farming. Whether we shall yet see a county agent for business to match the county agricultural agent is a matter for speculation, but there is a trend in that direction. In this connection, the role of the county agent in the administration of local organizations of farmers for the purpose of controlling production assumes importance.

What we need to face is the fact that, in today's society, the elements which limit size and the factors which determine efficient competitive performance operate to limit the range of economic and social benefits to be expected from privately owned enterprises. It is possible that legislative reform can raise the ceiling on size,¹³ but it is doubtful that such reform would be sufficient to remove the serious technical obstacles. It would seem that only a program of subsidy would aid here.

The small farmer has learned that survival depends in part upon certain kinds of protection and subsidy from the national government. Federal subsidy puts a floor under farm earnings, provides an improved base for long-range planning, affects financing, and permits specialization. The question which the student of small business faces is whether this is to be the pattern of the future for small business, too.

The farmer has learned to live by decisions made by an organized majority and as a result has given up the freedom to make some decisions for himself. It might be that in this process we have taken steps toward an ultimate institutionalizing of agricultural production. Perhaps a rational small business program calls for such procedures. Yet, this would mean that capitalism had destroyed itself by making this necessary!

Is Economic Competition Doomed?

Any future effort at creating a national environment for small business, patterned around the farm policy, must take into account the rulings of the Supreme Court which destroyed the National Recovery Act. It is apparent that such programs would call for basic modifications of traditional free-enterprise theory.

In any discussion of improving the economic climate by such means we must ask, does any individual have the inherent right to social mobility, to choose the way in which he wants to provide his livelihood to such an extent that he may legitimately demand the aid of government in fulfilling his ambitions and choices? If he does, then the basic rationale supporting a free market is doomed, for that rationale rests on social, not individual benefit. While the issue is present in our farm program, it would be more dramatically extended in any plan, patterned after the farm program, to aid small business.

What is demanded in all such programs is the prior determination of the social value of the way of earning a livelihood and the worth of the ensuing product. It is invariably in the name of community benefit that such privilege is sought and created. In this connection, we may suspect that it may be socially desirable to preserve a surplus of agricultural capacity and still not be equally desirable to preserve a surplus of small business enterprise.

Sociological questions here become more important than the purely economic ones. But the economic elements cannot be completely ignored, especially if small scale operations are functionally required to optimize the potentials of bigger enterprise. Yet, there is some indication that an assumption of necessity here is at least questionable. Quite often it is the psychological impact of having competition which is stressed in the arguments for promoting formation of new enterprises, particularly when the tendency of large-scale aggregates to stagnate is under discussion. But this tendency toward stagnation might be counteracted in other ways as well.

In part, too, the social value of small business depends upon the range of functions in a community which can be performed by an able leader who is independent because of business ownership, and who attempts to live up to the highest role he thinks the community expects of him. But such social value is a function of the role assigned to local communities in the total political structure of the nation, and to the importance given to alliances of local pressure groups on a national basis.

Structural considerations of this type are ultimately determined by social necessities as well as social ideals. Discussion of the role of small business is thus a part of the general debate over the nature and definition of both social necessities and social ideals, a debate of special importance to our age.

One significant difference between the operations of small business and of farms requires special attention. To a large degree, the present farm program has been shaped by organized political action in support of farmers. It does not seem that history will repeat for privately owned business.

The inability of small business associations to provide an ideological format for building effective political action is striking and it is not altogether explained in terms of the financial contributions made by big business to some of these associations. It appears as if lack of agree-

ments on a broad set of goals is due to the variety of factors which determine the social importance of privately owned enterprises, as well as to the variety of factors determining to what extent such social value is maximized or destroyed.

Clarification of the generalized and confused image of small business now being used is needed. Quite often it appears to have no more subtlety than is required of a model useful for explaining the routine functions of bookkeeping. The concept stereotypes inherited from the past require reexamination, but the factual material is hard to come by. It is to be hoped that studies sponsored by the Small Business Administration will remove this area of ignorance.

For the lack of a readily recognized common denominator drawn from the fields of the behavioral sciences rather than the economic studies, there is not an identifiable privately owned business viewpoint useful for intelligent political action. The connection to the agrarian ideals of a century ago is not enough to promote effective political action either on the part of members of the small business community or for their benefit on the part of others.

Outlook for Change

In the writer's opinion, this does not mean that there cannot be a viewpoint adequate for today; only that its creation calls for an essentially intellectual approach, an approach which boils down to an effort to determine, in Professor Galbraith's phrase, the nature of the "countervailing power" which ought to be vested in such individuals in a democratically oriented society, along with a program to secure a general public acceptance of such a definition.

Even if a new economic philosophy were available, a needed change in distribution of effective political strength cannot be achieved without some change in the political outlook of small businessmen. If small business is to be politically important, realism compels us to see that this will require a coalition of interests, in which the support of others is elicited in the form of political payment.

We can only conclude that the likelihood of any significant change in the social role of small business rests on the possibility of a fundamental shift in the political philosophy of businessmen so as to permit an open alliance with those social elements which have consistently fought for more rather than less government action in the social sphere.

The shift required is quite great, since there remains the old tradition of business dominance as the "natural state" of affairs. There is further the anti-tax attitude of businessmen and the greater fear of alliances which might promote ends now openly supported by labor union political leaders in the social welfare programs. The increasing use of sales taxes might induce some change here.

But against this, business leaders in the privately owned segment of the economy must realize that they have a great political handicap since

business bias is believed to prevent them from sharing any interest in the issues which stir their fellow voters.¹⁴ The present result is that independent businessmen in metropolitan communities are isolated politically and can easily be disregarded because of their small ballot impact. In fact, a "business blessing" may even be a political disaster to a candidate in some areas.

It is not very likely that traditional business attitudes will do more than shift on a temporary basis. Thus, it is also improbable that any significant changes in our economy as the result of political action will be traceable to the influence of small businessmen as innovators. If the interest in small business is political, it is not because of the pressure originating from the small businessman's hunting for "a better shake." It is now essential to uncover the direction from which it might come.

In Conclusion

The periodic political interest of liberals in small business functions may be regarded as a part of an effort to alter the present power distribution in the nation -- more narrowly, as an effort to provide obstacles to its further extension. It can be seen that this is a symptomatically oriented approach calling for the development of localized political support if it is to proceed further, a support based on some sort of "packaged program", rather than one specifically oriented toward small business, but which nevertheless commands selfish support from small businessmen.

Why is there such an interest? Primarily, I suspect, because no short-term program to bring big business under more direct public control can hope to succeed without the support of political leaders influenced by small town businessmen and drawn from the less populated areas of the nation.

The moot question for political leadership interested in social reconstruction is whether proposed legislation triggers off emotional response among the independent businessmen, a response separated from and able to withstand the attacks of business leaders drawn from the publicly owned corporations interested, for their own reasons, in preventing any ideological cleavage in the business community. This political tactic is then a part of the small business picture of today. Despite the political overtones, however, it is obvious that there are other reasons for seeking to develop an ideological concept of small business.

We need a concept if for no other reason than to form the basis for an intelligent understanding of public owned business entities, and especially the large-scale entities in this group. The process of forming such an ideology has been going on since the days of the original anti-trust legislation. The development of a "business conscience," the search for a useful definition of small business by Congressional leaders are only the latest steps in this process of providing a philosophy for legislative and judicial action.

It is not only that a lack of such a concept may injure small business; it is also that we are unable to understand intelligently the complex problems of big business unless we begin with a useful ideology. The assumption that the goals of all business are uniform is a dangerous simplification in today's international economic and social world.

In this process of determining the social role of big business it would be useful to have a framework providing an ideological socialization of small business. But it appears as if this must follow the effort to create a political front, in the name of small business, by professional politicians rather than precede such an effort.

What is most obviously at issue is the determination of the means to be used in controlling and directing the development of the economy. The attempt to link big business with small business can be understood as an effort to prevent any ideological division of the business community, a division which might promote further regulation of publicly owned business entities.

It is evident that the "small business problem" is both political and ideological. As presented here, the nature of the problem is such that the entire business community can be expected to be affected by the steps taken on both fronts. It involves the future of both large and small business, and it certainly involves the commercial relationships between these segments of the economy.

As such, I believe this problem to be a symptom of the lack of an economic philosophy directing the "economizing goals" of political society -- an expression of an uneasy concern over the feeling that day-to-day adjustments are shaping the future with a consequent loss in the sense of direction. Old ideals are mutilated in the process, and along with this there is a destruction of yardsticks by which a man determines his position in the political life of his community and identifies his natural and permanent allies. The perennial gale of destruction needs to be followed or accompanied by creative reconstruction.

Without such reformulated yardsticks, and the loyalties they create, the evolution of a society of masses seems inevitable. The foundation for a defense here is undermined, the more the social base is seen as a thing of shreds of old doctrine mixed up with patches of amendments. Thus, the "small business problem" involves far-reaching issues, and more serious consideration than it is likely to receive at present especially if it is to be regarded only from the narrow viewpoint of economics. But unless the senators quoted at the beginning of this article are wrong, we can expect to hear more about the problem in the future, and we can expect to see deeper interest in the matter in business circles.

NOTES

1. Opening sentence of the Seventh Annual Report, 85th Congress, First Session.
2. No less than the Assistant Secretary of Commerce for Domestic Affairs, testifying before the House Banking and Currency Committee, 85th Congress, First Session, is recorded as having said that the only reason for having a separate small-business agency "is (as a) political gimmick which probably requires that we have an administration (small business) that is giving all its attention to the problems, so-called, of so-called small business."
3. Max Lerner, America as a Civilization, Simon and Schuster, 1957, p. 345.
4. This difference is evident in the various annual reports of the Select Committee on Small Business and in the reviews of the reports of the Small Business Administrator conducted by the Senate Committee. In its Eighth Annual Report (1958), the Committee charges, page 15: "The agency has maintained an almost complete dependence upon established, congressional-dictated operations. No bold, new assaults on the myriad problems of small business have been charted by the Small Business Administration." The report goes on to add, "Even more disappointing has been the largely negative reaction of Small Business Administration spokesmen to proposals made by others."
5. Authors of "The Competitive Struggle for Marketing Control," The Annals of the American Academy of Political and Social Science, May, 1940, pp. 84-107.
6. Businesses can of course differ in their political effect on a society. This is ignored in the present discussion. But the groundwork for studying this effect has recently been set forth by Robert A. Dahl in "Business and Politics: A Critical Appraisal of Political Science," American Political Science Review, March, 1959.
7. From A Study of the Development of Anti-trust Laws and Current Problems of Anti-trust Enforcement, a 1952 report prepared by the Department of Justice for the use of the Select Committee on Small Business, U.S. Senate. The report adds that a claim has been made that "... a healthy anti-trust enforcement policy and a more competitive economy in the years preceding the depression would have prevented it altogether."
8. Ordway Tead, The Art of Administration, McGraw-Hill, 1951, p. x.
9. Ownership of property is not necessary even to manufacturing success as pointed out by the author in "Manufacturing in Jeopardy," Harvard Business Review, November-December, 1959.

10. With the multiplication of jobs requiring formal education and the enlargement of opportunity for gain through such education, the notion of "equality of opportunity" is considerably strengthened. Robert E. Lane, in "The Fear of Equality," American Political Science Review, March, 1959, asserts: "The greater the emphasis in a society upon the availability of 'equal opportunity for all,' the greater the need ... to develop an acceptable rationalization for their own status. The greater the strain on a person's self esteem implied by a relatively low status in an open society, the greater the necessity to explain this status as 'natural' and 'proper' in the social order." (Page 49).
11. The paucity of reliable data is probably a major factor in delaying further congressional action in the name of small business. In its annual reports the Select Committee on Small Business has stressed this lack of statistical data, and the Eighth Annual Report indicates that the Committee intends to have the Census Bureau provide more information on small business.
12. This unpublished survey is reported in part in The Small Manufacturer and His Specialized Staff, written by Donald R. G. Cowan for the Small Business Administration as Booklet No. 13 in the Small Business Administration series.
13. Such reforms would necessarily take the form of changes in taxation methods affecting owners of businesses. There were some steps in this direction in 1958.
14. The usual reasons for urging businessmen to political activity show distinct fear of labor's political power. This is evident in Horace E. Sheldon's "Businessmen Must Get into Politics," Harvard Business Review, March-April, 1959; and in William L. McGrath's "The Political Responsibilities of U. S. Businessmen," Advanced Management, August, 1959.

CARTELS, COMPETITION AND PUBLIC POLICY IN GERMANY

Theodore Marburg*

Economic issues involved in assessing the effects of anti-trust policy in the United States were explored in the December issue of the Marquette Business REVIEW in an article entitled, "Competition, Business Size and Economic Performance." In that article, economic performance was shown to depend in significant measure on innovations which contribute to lowering of costs, product improvement or new products. Our anti-trust policy, however, was shown to be directed toward the maintenance of competitive structure, on the premise that competitive structure contributes to optimum performance. A comparable consideration of German policy will be undertaken in this article, which will close with consideration of the 1957 Cartel Law and the further legislation that is currently proposed.

Two issues were explored in the earlier article on which there are, in America, widely differing judgments and policy recommendations. On both issues, there was a marked difference between the prevailing economists' views here and in Germany. On both issues, however, there are some common approaches that bear exploration.

The first such issue which we face in the United States today, and which was explored in the December REVIEW, turned on the degree to which trade association rules, spelling out terms of business conduct and rivalry, may serve to maintain competition even though restricting some particular phases of competition or tactics in competition. Contrary to the prevailing trend of American court decisions and economic analysis, which condemn price fixing or market sharing per se, there has been some favorable comment. In exceptional circumstances, the courts have upheld restrictive rules on pricing procedure and a common sales agency with division of market. Thus, there are: Chicago Board of Trade vs. U.S., and Appalachian Coals, Inc. vs. U.S. Furthermore, the industry wide use of a common basing-point pricing system was not condemned by the Supreme Court until the late 1940's, and it has been articulately defended by such an economist as J. M. Clark. This line of reasoning

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supporting trade association activity can be related to pre-war German views on cartels.

The second issue explored in the earlier article turned on the degree to which the size of the firm and its market dominance may contribute to conditions bringing on innovation in product or in production process. Our law expressly calls for prohibition of restraints and of monopoly, regardless of the constructive performance of large or dominating firms. A study by John Jewkes suggests, also, that new products enter from outside large firms. Yet an articulate body of economists in the United States, led perhaps by Joseph Schumpeter, have taken the position that much progress comes from the large firm, and that this progress is of more significance to the economy than optimum operation at any given time. This line of reasoning finds a counterpart in early German views on mergers, trusts or combines.

The German legislation generally cited by the courts on cartel cases was passed in 1869 and 1871, before cartels in their modern form had achieved widespread notoriety. It had reference to freedom of contract in general terms. Later legislation on a related issue was a 1909 statute on unfair competition which, together with several later laws, constituted a body of legislation distinct from that on cartels. Government pressure was exerted affirmatively in 1910 in support of a cartel in the potash industry and in 1919 in support of a cartel in the coal industry. A 1923 Ordinance Against Abuse of Economic Power spelled out a number of requirements for valid cartel contracts, and provided for termination of contracts found contrary to the public interest. In the 1930's there was further legislation which broadened the powers of the Reich Minister of Economics. By an Act of 1934 he was given general power to compel membership in cartels. There was thus, in the years before the second World War, government countenance of cartels to a degree that provides a marked contrast to the main lines of American anti-trust policy.

The statute cited in cartel cases of the late 19th century was the Trade Regulation Act for the North German Confederation of 1869, adopted as a Reich statute in 1871. This declared, in a general clause of Section 1, that trade is open to all parties, except as such freedom is limited by statute. Freedom of contract was, in effect, specifically declared to be public policy. A major case, one concerning an association of woodpulp manufacturers, was decided in 1897. In the decision, the court held that the cartel contract did not violate the general clause of the Trade Regulation Act, and furthermore that the cartel contract did not impair personal liberty in a manner contrary to the statute.¹

The decision can be considered representative of those rendered in pre-World War I years. Statutory amendments did not significantly alter the pattern. There was a statute on competition of May, 1896. In June, 1909, the legislature passed the Law on Unfair Competition. Once again it was the general clause which proved pertinent. This provided that anyone conducting competitive trade in a manner contrary to good morals could be sued for damages. The pertinence for cartel participation rested on the generally accepted role of cartels. As one

analyst suggested, the cartel violator could be regarded as more seriously violating good morals than the cartel member who abided by his contract.

Cartels received attention in the legislature in the late 19th century. In 1879 Eugene Richter delivered a speech in the Reichstag in which he pointed to the fact that German railmakers and locomotive manufacturers were charging higher prices on the home market than they were charging abroad. Perhaps the rise of tariff protection in Germany after 1879 made restrictionist practices worthwhile and their consequences evident. Early in the century there was a legislative inquiry into cartels. In 1905 an inquiry was undertaken by the Ministry of the Interior which tallied 385 associations, although this figure may be low. The operating cartels, mostly in manufacturing, were tallied by the Federation of German industries in 1923 and the total came to over 1,500. A more complete government count of two years later placed the number at about 3,000.²

The large number of cartels that were established in Germany reflects the prevalence of this organizational form. It is worth noting, however, that this number does not, in any degree, reflect power of cartels. One reason there were so many cartels was that few cartels embraced all manufacturers in an industry on a national scale. Some cartels were regional. Some were for manufacturers that could not be classed as representing an industry. In the toilet paper industry, for example, there was a special cartel for the manufacturers of crepe toilet paper.

The cartels were also subjected to scrutiny by a number of economists. Most of the writers took a sympathetic view. Friedrich Kleinwachter published a volume in 1883; Robert Liefman one in 1897; and in 1904 S. Tschierschky compiled and published a bibliography of cartel literature.³

A cartel was generally defined as an association of firms which retained their financial independence but undertook cooperative efforts to regulate market conditions. Critics viewed the cartels as seeking to achieve monopolistic restriction of output. Advocates of cartels, who were more numerous and articulate, held that by providing order and stability in the markets, cartels contributed to general stability and growth.

There was a scholarly periodical, *Kartellrundschaу*, started early in the century, which reviewed policy issues and their implications. Among the numerous articles by S. Tschierschky of Dusseldorf, there was one in 1909 which pointed up the difficulties in carrying out a public policy that was neutral with respect to cartels. The Chamber of Commerce of Kassel had refused to support the re-establishment of a particular cartel. Tschierschky held that a body such as the Chamber, obliged to serve the whole community, might well find itself subject to conflict of interest if it supported a cartel which sold to purchasing firms that were also members of the Chamber. The logic of the position, however, was the more striking since most chambers aggressively supported a variety of cartels, for instance that of Essen came to the defense of the coal cartel.⁴

The German economists watched the contrasting development of the anti-trust policy in America with much interest. Following the passage of our Clayton Act in 1914, Robert Liefmann published a comment in the Quarterly Journal of Economics. The American preoccupation with preventing mergers or agreements seemed to Liefmann misdirected, formulated without full cognizance of the advantages of large size. "... American policy," he held, "endeavors in every way merely to maintain the competitive struggle."⁵

This first phase of German policy has been aptly characterized by Franz Boehm, an economist of the neo-liberal persuasion who is also currently a member of the Bundestag and who suggests that in the early period:

Economic freedom came to mean merely the freedom of the economy from the State, not, however, the freedom of the economy from the interventions of private cartels and monopolies. According to this view, economic freedom includes the freedom of business men to limit, or even eliminate competition, as well as their freedom to hinder other business men from competing, and to force them to join cartels or to sell their concerns to monopolistic large enterprises; it would, on the other hand, represent an infringement of economic freedom were the State to prohibit or hinder the conclusion of agreements in restraint of competition or the use of boycotts.⁶

The development of compulsory cartels in special cases, as in potash, and in coal mining under an act of 1919, may be regarded as a climax to this period.

* * * * *

A first step toward statutory provision for restraint of cartel abuses was taken with the Ordinance Against Abuse of Economic Power in 1923. Statutory provision was necessary before there could be any shifting trend on court decisions, since the system of civil law which prevails in Germany allows less latitude for changing patterns of interpretation than occurs under our system in the United States.⁷

The Ordinance of 1923 became law under unusual circumstances: the legislature had authorized the government to establish a new currency and terminate the planned economy. The government issued various orders which had the power of laws and which included this particular order of November 2, the first explicit Cartel Law.

This Ordinance replaced the prior system of cartel freedom, subject only to judicial limitation in exceptional cases, by a system of administrative agency control. All cartel agreements were required to be in writing. The Reich Minister of Economics was empowered to present to a newly created Cartel Court evidence showing that a cartel agreement should be voided. Members of cartels could, under certain circumstances, be released from their contracts. Section 9 provided that the imposition

of penalties by cartels and the posting of bond to assure compliance was allowable only with the consent of the president of the Cartel Court.⁸

The action taken under the 1923 Ordinance was limited in scope and extent. So much so, that in 1926 the Social Democrats proposed that there should be a continuing Cartel Register, a permanent Cartel Office, and annual inquiry into the number and effects of cartels. From other sources came support for a cartel inquiry in 1930. A few cases were nonetheless brought, particularly under Section 9 of the Ordinance. This section prohibited the imposition of penalties. Thus action was instituted upon complaint of businesses that had been boycotted. Professor Boehm has suggested, in retrospect, that the clause was just beginning to have wider meaning in the early 1930's, when its further use ceased due to the change in over-all policy.

An Emergency Ordinance to Meet Financial, Economic and Social Emergencies was issued July 26, 1930. This broadened the power of the Reichminister of Economics to interfere directly with cartels. In 1933 a further amendment to the 1923 Ordinance was drafted empowering the Minister to act directly, rather than through the Cartel Court, in voiding cartel agreements found contrary to the public interest. This amendment was enacted into law under the National Socialists in 1934 but was never used.

In 1934 the Law for Compulsory Cartels was also passed, and this proved to be significant in the following years. After the price freeze of 1936, the government utilized the cartels in a semi-official capacity. This added to their prestige. In one sense, this government reliance upon the existing industry organizations can be compared to our own, similar, reliance in the United States during both World Wars, and in the depression code-making period under the National Recovery Administration.

For American readers, the code authorities of the NRA period afford a familiar example of the opportunities and dangers in compulsory cartel organization. The activities of our NRA code authorities also provide a suggestive illustration of the range of activities likely to concern such organizations when, as was the case, anti-trust rules are suspended. A small number of the code authorities restricted new entry, the restriction being tightest in lumber. A larger number undertook price maintenance. Many more concerned themselves with restricting competitive tactics that were regarded as destructive. Such activities of the NRA code authorities and the German Compulsory Cartels can also be compared with the governmentally sanctioned industrial self regulation in England of the same period.

The economic aspects of public policy were examined by economists with renewed vigor. The government was obliged to formulate criteria for action under the 1923 Ordinance and later legislation. This task gave pertinence to economic inquiry. The fruitful and wide-ranging studies of the period were surveyed in a bibliographic essay by Arnold Wölfer in the early 1930's.⁹ Wölfer points to the concern, in this period, as to the degree to which cartels furthered production, and the degree to which they operated in a restrictionist manner. These issues were

explored. A number of studies were later made available to the American reader in English, such as those by Liefmann, H. v. Beckerath, Levy and Pribram.¹⁰

A study prepared by Karl Pribram is revealing. The author had been professor of economics in Vienna and at Frankfurt. His study was prepared primarily for American readers, under the sponsorship of the Brookings Institution in Washington, as one of its series in relation to the American experience with the National Recovery Administration. Pribram outlines the similarities in the purpose between cartels and trade associations, but indicates that in American analysis, because of the anti-trust laws, we distinguish between association activity that is designed to control markets, forbidden under our law, and the wide range of allowable activity. His discussion of cartels is directed to associations which place restrictions on individual members' producing or pricing for the purpose of market control.

Within this framework Pribram explored such controversial issues as "whether stabilization of prices as attempted by the bulk of strong German cartels ... is apt to stabilize economic conditions."¹¹

Analysts supporting cartels base their position upon an affirmative answer to this question. Yet economic reasoning leads to a negative conclusion. Rigidity of prices in the face of altered demand would lead to a widening change in output and levels of employment.

In 1933 two studies by Professor Boehm appeared in Germany which are today regarded as providing an economic rationale for the neo-liberal approach to the cartel issue.¹² They dealt with competition and cartels. Both of these studies were given major attention by German economists in the period after World War II.

The issue of German cartels came conspicuously to the attention of the American public in connection with judgments on the source of Germany's war potential, and in relation to policy recommendations for the disarmament of Germany. In loose parlance, cartels came to mean both cartels proper and large firms such as I. G. Farben.

It was on this basis that the general decentralization policy was set forth in the Report on the Tripartite Conference of Berlin of August 2, 1945. In Section III, paragraph 12, there is the declaration:

At the earliest practicable date, the German economy shall be decentralized for the purpose of eliminating the present excessive concentrations of economic powers as exemplified in particular by cartels, syndicates, trusts and other monopolistic arrangements.¹³

In the spirit of this declaration, three comparable laws were put into force in the United States, the British and the French Zones. In the United States Zone, U.S. Military Government Law No. 56 of February 2, 1947, bore the title "Prohibition of Excessive Concentration of German

Economic Power." The Allied High Commission established the Decartelization and Industrial Deconcentration Group, D. I. D. E. G., which proceeded to issue cease and desist orders.

The actions of the D. I. D. E. G. were never regarded by the Germans as being aimed at achieving dynamic competitive conditions in Germany. They were, as Professor Boehm points out:

... universally considered as penal actions and demilitarization measures in the same category as dismantling... intended to harm Germany's competitive position in world markets.¹⁴

Allied policy in carrying out deconcentration, which failed to emphasize the distinction between association activity and large firm dominance, is reported by Walter Damm to have "caused substantial confusion among German economists and lawyers who had always made a careful distinction between ... cartels ... and trusts."¹⁵ Concern over this confusion and over the question of whether the purpose of deconcentration was to weaken Germany or make the economy more dynamic is reflected in a wide range of writing by German economists.¹⁶

The American Law No. 56 and the British Law No. 78 both declared that cartels and combines constituted excessive concentration of economic power. In the law and on the declaration, cartels and combines were both condemned in the same breath. In practice, it was necessary to proceed differently in the two classes of cases and a distinction emerged.

Cartels and associations achieving cartel purposes were prohibited except as the implementing agency granted exceptions. All such organizations were required to file statements making their function and activities a matter of record. The implementing agency could then issue an order to the cartel directing it to show why it did not fall within the terms of the Law. Hearings might be granted, but in the early years orders were issued without hearings. The agency then determined whether the cartel was exempted or in contravention of the Law. In the latter case, a cease and desist order was issued.

Allied activity with respect to cartels and other associations which restricted entry was carried out with such effect as to actually fire up entry into a wide range of business lines. During General Lucius Clay's service as Military Governor, he directed that local governments provide for free entry of refugee-craftsmen into their line of work. This directive ran contrary to earlier German practices, under which entry was contingent on a finding as to community need, and it has had significant effect on present statutes and decisions.¹⁷

Large single firms employing more than 10,000 persons were likewise required to file. The mere fact of their size as reflected in this number of employees was regarded as prima facie evidence of excessive concentration. But the practical problems of breaking up a large single enterprise were great. Thus, the implementing agency undertook a case-by-case examination of the large single enterprises

before ordering divestiture or dissolution. The conspicuous and significant cases of deconcentration under this law included action in the movie industry, the chemical industry in action on I. G. Farben, and the electrical equipment industry in action on Robert Bosch. Comparable action in banking and in iron and steel was achieved under distinct legislation not covered in this survey, but it is worth noting that in these industries re-concentration was subsequently approved, on grounds of economic or business necessity. It appears that the breaking up of large firms was not given full support in top councils of Military Government and the later High Commission. General Clay, himself, in his retrospective account, points out that he regarded the breaking up of large firms as a less clearly desirable goal than prohibition of cartels.¹⁸

It may well be that the limited scope of deconcentration action in the cases of large firms was due to changes in American policy that resulted from the emergency of the cold war and recognition of the importance of Germany as an ally. Any deconcentration, which clearly reduced productive potential seemed inappropriate under the altered circumstances.

Occupation authorities continued to retain the deconcentration powers over the German economy under terms of a 1949 directive. In 1951 they indicated such powers would be relinquished when satisfactory legislation had been established in Germany. Under terms of a 1952 agreement, the Allies agreed to relinquish such powers as soon as legislation was passed with substantive provisions such as those of the bill then before the legislature. At a 1954 foreign ministers' meeting, however, the Allies relinquished such powers unilaterally, the German government agreed to seek passage of a law that would contain "general prohibitions against restraints of competition." It had, however, been released from an earlier obligation to enact an outright cartel prohibition law.¹⁹ The Allied powers in this matter were formally terminated by the Bonn conventions of 1955 which declared that the deconcentration laws then in effect would be in force until repealed or replaced by the German legislative bodies.²⁰

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Ludwig Erhard has proved a most articulate exponent, in postwar Germany, of market freedom within the framework of the social economy. In 1948, he had a major part in developing a program for the removal of rationing at the time of the currency reform. Subsequently he appointed a committee which submitted, in July, 1949, a first draft law on restraints of competition. This first draft was essentially a cartel prohibition law, but with specified exceptions.²¹

Support for the major parts of Erhard's program came from a wide range of sources. Ideological support for the objective of freeing the economy came from such economists as Boehm, already mentioned, the late Walter Erickson, and others. The constructive role of competition was emphasized in writings of members of the so-called Freiburg School. Analysis of issues in this framework has been made available in the annual Ordo founded by Eriksen and Boehm and now edited by Boehm, Friedrich

Lutz and Fritz Meyer. The views of these economists on the appropriate role of competition constitute only one facet of the wide range of policy measures embraced under the term neo-liberalism. Kronstein suggests that the enactment of the statute "is the expression of an ideological movement aiming at a free competitive order."²²

A chief tenet of the neo-liberal view is that appropriate institutional patterns may create a structure within which wide latitude of economic freedom can contribute to furtherance of effective economic performance. Thus the enactment and administration of a constructive and effective law on competition serves to illustrate the problems in implementing the policy of neo-liberalism.

In 1951 a government draft of a cartel prohibition law was accepted by the Cabinet and in 1952 it was submitted to both houses of the legislature. Here the bill encountered such strong opposition that it was withdrawn and reconsidered by a working party consisting of representatives of the Ministry of Economics, the Ministry of Justice and the Federation of Justice and the Federation of German Industries. The bill that emerged from these discussions has been characterized as having "practically eliminated the prohibition principle from the Government Bill and [as having] introduced the abuse principle."²³

A weaker bill, more acceptable to the Federation of German Industries, was then introduced by H cherl in 1955. A third bill reaffirming the prohibition principle was introduced by Professor Boehm in the same year. Finally, in 1957, the government bill, as altered, was passed. On this side of the Atlantic it was acclaimed, in the *New York Times*, as a "big victory for Erhard over industrial leaders." The *London Economist*, however, characterized it as "Bonn's Elastic Cartel Law."²⁴

A translation of the 1957 Law may be easily obtained from the United States Bureau of Foreign Commerce. Commentaries are available in the American law journals and several book length commentaries have been published in Germany.²⁵

The economic significance of the Law may well hinge on the way the administration interprets the provisions expressly authorizing cartels.

The major impact of the Law will be with respect to horizontal cartels, or restraints among manufacturers of the same product. This involves restraints of the sort prohibited in this country under Section 1 of the Sherman Act. Such cartels are prohibited unless they can qualify under one or another of seven special categories. The Law also recognizes the problem of the market-dominating firm. This is the issue covered in the United States under Section 2 of the Sherman Act and under Section 7 of the Clayton Act as altered in 1951 by the Kellar-Kefauver amendment.

Administration of the Law is to be by a newly created Federal Cartel Authority, under the jurisdiction of the Minister of Economics, and by

the Minister directly. Approval of cartel arrangements that are allowable under the law is by this Cartel Authority or directly by the Minister of Economics, but any such approval is normally not to be granted for more than three years; the cartel arrangement is then subject to surveillance and permission may be revoked, restricted or conditioned. Surveillance of market-dominating firms and recording of mergers is also a function of the Cartel Office.

Cartel-type associations may be permitted or specifically authorized in a wide range of cases. The prohibition and the exemptions are all included in Title I, Chapter 1 of the Law. Section 1 states the prohibition and Section 2 indicates that associations which establish uniform terms of delivery and discount must file documents showing such terms, and that these may then go into effect if the Cartel Authority does not enter objection within three months. Section 3 provides exemption for cartels specifying uniform rebates, provided the basis for granting a rebate does not lead to unjustified discrimination and provided documents describing such bases are filed with the Cartel Authority and are not objected to within three months of filing.

Section 4 of Title 4 provides for allowing cartels a wide range of activity in event of an industry crisis as may be brought on by decline in sales due to a lasting change in demand. This is designated the "crisis cartel" provision. These are to be compared with Appalachian Coals Company, Inc.

Section 5 provides grounds for licensing of rationalization cartels. Those which deal exclusively with the uniform application of standards may be sanctioned in the same manner as cartels covering uniform terms of sale. Agreements must be filed with the Cartel Authority and become binding if the Authority has not objected within three months.

Rationalization cartels that undertake price agreements and joint purchasing or marketing organizations must obtain specific license from the Cartel Authority. License to carry out such cooperative activity is to be granted only if the rationalization is in the public interest and cannot be achieved by other means. The Law requires, further: "The restraints of competition must be reasonable in relation to the rationalization effect."

Section 6 provides authorization for export cartels and Section 7 for import cartels. Special provisions are intended to harmonize the law with the terms of the Coal and Steel Community and the emerging program of the European Common Market.

Section 8 authorized the Minister of Economics to license cartels without prior action by the Cartel Authority. The law is quite explicit in limiting cartel licensing under this clause of cases of immediate danger for which other remedies cannot be authorized in time.

Market-dominating firms are covered in Title 1, Chapter 3 of the Law. A firm is considered to be market-dominating if it has no competitor or is not exposed to any substantial competition in any particular

market. Such a firm is subject to action by the Cartel Authority, if at all, only after there has been a finding that the firm is abusing the power it holds by reason of its market dominance.

Market dominating firms that are found to have abused their market power may be ordered to refrain from such activity. Contracts may be declared invalid.

Mergers and intercorporate transfer of assets must be recorded with the Cartel Office if the resulting merged enterprise will control 20 per cent of the relevant market. The Cartel Office has what we may lightly designate as powers of "exhortation" with respect to such mergers.

Some tentative lines are appearing in the emerging pattern of practice and control under the 1957 Law. A number of cartels have been licensed under the terms of Section 5, which authorizes rationalization cartels, and mergers have taken place that have increased market power of dominant firms.

At the time the Law was passed there were grounds for expecting that the provisions in Section 5 could be used to justify re-establishment of cartels in a variety of industries. One example of a rationalization cartel with joint marketing, etc., was the Steinzeug-Handellsgesellschaft, a common sales agency through which 70 per cent of the relevant manufacturers undertook to channel their entire sales, with heavy penalty imposed on any firm making outside sales. The Minister of Economics licensed this association in May, 1957, in spite of the fact that the marketing agency fixed prices and allocated sales on the basis of capacity. The basis for granting a license in this case rested on the premise that centralization, and in effect, rationalization by specialization, was in the public interest and could only be achieved through proper distribution of orders by means of a common marketing agency.²⁶

Since the Law went into effect, a number of cartels similar to the Steinzeug cartel have been licensed. This particular cartel has been reviewed again under the terms of the New Law and has been cleared.²⁷ A comparable arrangement has been cleared for the cartel in the potash industry.

Eugen Langen has pointed out that the basis for authorizing such a cartel rests in part on a plan of action for rationalization that is to take place in the future. This leaves a wide range of cases in which the decisions now being made appear to be reached on the basis of declared intentions. This will cause future difficulties.²⁸

Administration of Section 5 makes necessary regulation or supervision. The provisions of the Steinzeug license, for example, call for annual review in a manner designed to show whether the rationalization objectives are being achieved. There is the possibility that the economy may thus come under regulation as a result of legislation intended to implement a program of economic freedom.²⁹

Meanwhile, the growth of large firms has received little attention. The Cartel Authority has a very limited role in surveying or remedying such degree of one-firm market power as would be considered market dominance in American thinking. Single firm expansion and mergers are subject to the law only where there is extreme market power and where it is abused. Concern for the survival of small business is expressed and gives grounds for concern over degree of market dominance.³⁰

In October, legislation was proposed in the Bundestag by both the Administration and the opposition party. The more extreme legislation, proposed by the opposition, is that a permanent, independent Monopoly Commission be created. This commission would oversee business operation and inquire into market dominance defined somewhat broadly.

The government proposed two measures. The first was that there be an official inquiry into the degree of economic concentration. The second called for an inquiry as to whether existing laws need be altered in view of their effect on mergers, concentration or abuse of economic power. It will be interesting to observe what action is taken on those proposals. If an inquiry as to degree of concentration is instituted it will provide data for comparison and contrast with American structure.³¹

Neither of the current proposals calls for inquiry as to the relation between structure and performance. In this respect, the Germans are in the same dilemma as we. The present state of our discipline of economics does not provide criteria for measuring relative effectiveness of performance of industries under conditions of dynamic change.

NOTES

1. This case is cited in Ivo E. Schwartz, "Antitrust Legislation and Policy in Germany - A Comparative Study," University of Pennsylvania Law Review, Vol. 105 (March, 1957), pp. 617-690. Schwartz comments on this case, pp. 626-635. His citation is: B. von den Saechsischen Holtzstoff-Fabriken-Verband, Reichsgericht (VI Zivilsenat), Febr. 4, 1897, 38 T.G.Z. 155.
2. R. Liefmann, Cartels, Concerns and Trusts, London: Methuen, 1932, see pp. 25, 30, 31.
3. Friedrich Kleinwaechter, Die Kartelle, Innsbruck: Wagnerverlag, 1883; Robert Liefmann, Die Unternehmerverbaende, Freiberg: Mohr, 1897; subsequently Liefmann wrote Kartelle, Konzerne und Trusts which went through many successive editions and was translated into English and published as Cartels, Concerns and Trusts, London: Methuen, 1932; S. Tschierschky, "Ein Bibliographie der Kartellliteratur," Kartellrundschaue II (December, 1904), pp. 860-864.
4. S. Tschierschky, "Handelskammern und Kartelle," Kartellrundschaue, VII (March, 1909), pp. 227-248. This author refers the reader to his own volume of 1905 that explores the issues more fully: Die Organization der industriellen Interessen Deutschland.

5. R. Liefmann, "Monopoly or Competition as a Basis of Government Trust Policy," Quarterly Journal of Economics XXIX (February, 1915), pp. 308-325, see p. 318.
6. Franz Boehm, "Monopoly and Competition in Western Germany," in Edward H. Chamberlin (ed.) Monopoly and Competition and their Regulation: Papers and Proceedings of a Conference held by the International Economic Association. London: Macmillan, 1954, pp. 160-161.
7. The significant differences between the German and American systems of law and court interpretations are not discussed here. They are described with reference to the Cartel Law in the article by Ivo Schwartz, *loc. cit.*, and in Heinrich Kronstein, "Cartels under the new German Cartel Statute," Vanderbilt Law Review, XI (March, 1958). A Symposium on Trade Regulation and Practices, part 2, pp. 271-301.
8. This Ordinance of 1923 is succinctly summarized and evaluated in the chapter by Franz Boehm, *loc. cit.* The full text is reproduced in Liefmann, Cartels, Concerns and Trusts, pp. 351-357.
9. Arnold Wölfers, "Das Kartellproblem in Lichte der deutschen Kartell-literatur," Schriften der Vereins fuer Sozialpolitik, CLXXX, part 2 (1931). Among the writers listed by Wölfers are Tschierschky, Lehnich, H. v. Beckerath, Schmalenbach and Halm.
10. R. Liefmann, Cartels, Concerns and Trusts, *op. cit.*; H. v. Beckerath, Modern Industrial Organization, New York: McGraw-Hill, 1933; Herman Levy, Industrial Germany: A Study of its Monopoly Organizations and their Control by the State, London: Cambridge University Press, 1935; Karl Pirbram, Cartel Problems: An Analysis of Collective Monopolies in Europe with American Application, Washington, Brookings Institute, 1935.
11. Pirbram, *op. cit.*, p. 120.
12. Franz Boehm, Wettbewerb und Monopolkampf, Berlin: Heymanns-Verlag, 1933; also Kartelle und Koalitionsfreiheit, Berlin: Heymanns-Verlag, 1933.
13. Quoted from Ivo Schwartz, Antitrust Legislation, *loc. cit.*, p. 642.
14. Boehm, "Monopoly and Competition," *loc. cit.*, p. 148.
15. Walter Damm, National and International Factors Influencing Cartel Legislation in Germany, mss. Ph.D. dissertation, University of Chicago, 1958.
16. These issues were explored over pertinent years in the monthly Wirtschaft und Wettbewerb and in the annual Ordo. A later review was undertaken in 1957. Ernst J. Mestmaecker, "Dekartellierung und Wettbewerb in der Rechtsprechung der deutschen Gerichte," Ordo XI (1957) pp. 99-130.
17. Kronstein, "Cartels," *loc. cit.*, p. 273. Background on directives of the Military Government of November 29, 1948, to the four Laender of the U. S. Zone is provided in Monthly Report of the Military Governor, No. 41, November, 1948, p. 74.

18. Lucius DuB. Clay, Germany and the Fight for Freedom, Cambridge; Harvard University Press, 1950.
19. Damm, mss., op. cit., p. 9; Schwartz, "Antitrust Legislation," loc. cit.
20. Since deconcentration in banking and in coal, iron and steel, and the particular issues with respect to Krupp were covered by separate legislation, they are not covered in this survey.
21. Erhard describes the role of this committee, which included such professional economists as Professor Franz Boehm and Professor Bernhard Pfister, in his 1948 volume. See Ludwig Erhard, Prosperity Through Competition, London: Thames and Hudson, 1958), pp. 118 ff.
22. Kronstein, "Cartels," loc. cit., p. 273.
23. Damm, mss., op. cit., p. 10.
24. New York Times, July 5, 1957; (London) Economist, July 20, 1957.
25. "New Law on Cartels in the Federal Republic of Germany," World Trade Information Service Economic Reports, Part I, No. 58-1, Bureau of Foreign Commerce of the United States Department of Commerce, January, 1958. An outstanding short commentary for American use is that by Heinrich Kronstein, "Cartels," loc. cit. Among the commentaries published in German, the following are particularly noteworthy: Harald Rasch, Wettbewerbschraenkungen, Kartell- und Monopolrecht, Berlin: Verlag Neue Wirtschaftsbrieft, 1957; Eugen Langen, Kommentar zum Kartellgesetz, Nieuwied: Luchterhand Verlag, 1958.
26. Kronstein, "Cartels," loc. cit., pp. 285-286.
27. Eugen Lange, "Die Zulassung von Rationalisierungs- und Spezialisierungs-Kartellen," Wirtschaft und Wettbewerb (April, 1960) pp. 235-250.
28. Langen, "Rationalisierungs-Kartellen," loc. cit., pp. 249-250.
29. This view is suggested in industry publications. See, for example, Paul Riffel, "Die Einstellung der Industrie zum neuen Kartellrecht," Vortragsreihe des Deutschen Industrieinstituts, No. 41 (October 12, 1959; also, Anon., "Begrenzt das Kartellgesetz die freie Meinungsäusserung," Anlage zum Schnelldienst des Deutschen Industrieinstituts, No. 31 (April 22, 1960).
30. Guenter Henle, "Konzentration und Soziale Marktwirtschaft," Vortragsreihe des Deutschen Industrieinstituts, No. 4 (January 26, 1959). This issue, which deserves broader treatment than is possible in this article, has recently been broached in an article by the President of the Federal Cartel Authority. See Eberhard Guenther, "Das Gesetz gegen Wettbewerbschraenkungen als Instrument der Mittelstandspolitik," Wirtschaft und Wettbewerb X (March, 1960) pp. 159-163.
31. Ludwig Goldmann, "Gross Anfrage zur Wirtschaftskonzentration im Bundestag," Wirtschaft und Wettbewerb IX (December, 1959), pp. 860-866. For American comment see Egon Sohmen, "Competition and Growth, The Lesson of Germany," American Economic Review, XLII (December, 1959), pp. 986-1003. The current merger movement is discussed on p. 997.

LINEAR PROGRAMMING PROBLEMS SOLVED BY SYMMETRIC METHODS

J. Talacko and R. T. Rockafellar*

This is an elementary exposition of the new computational method of linear inequalities and other linear algebra. The Symmetric Method, developed mainly for Finite Matrix Games and Linear Programs, is a direct, pivotal, repetitive procedure. One simple algorithm, that is easy to learn, understand and apply, solves linear problems of all kinds in one Standard Form. Optimization processes of linear inequalities are reduced to elementary matrix manipulation, without any change on equations, without artificial variables and basic feasible solutions required. What is really new is the concept of algebraic representation of primal and dual weak-inequality systems by an abstract structure, which offer the direct solution of inequalities. This structure uses the bilinear formulation for identifications of coordinates and for possible geometrical or other interpretations at every stage of realization of the problem. The method is a really ideal computational technique of two-person zero-sum games, which are solved mechanically in a minimum number of iterations and in ultimately condensed size, for maximum and minimum at once.

Many advantages of the Symmetric Method are apparent, if applied on linear programming problems. Finite matrix games and linear equations problems are, practically, particular cases of linear programs.

I

The numerical solution of all linear programs proceeds through two stages. First the problem must be formulated into linear inequalities and then stated conveniently in maximum mixture problem (in a common matrix notation):

$$1. \quad Ay \leq b, \quad \max (cy');$$

where the matrix A is a real, rectangular matrix of the $m \times n$ order,

$A = a_{ij} ; i = 1, 2, \dots, m; j = 1, 2, \dots, n$; the $b = \{b_1, b_2, \dots, b_m\}$, and $c = \{c_1, c_2, \dots, c_n\}$ constraint vectors; $y = \{y_1, y_2, \dots, y_n\}$ the maximization vector.

Needless to say, a compound "activity matrix"

$$\begin{bmatrix} A & b \\ c & o \end{bmatrix}$$

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implies the corresponding dual-minimum problem

$$2. \quad A^T x \leq c, \quad \min (bx'),$$

of the minimizing vector $x = \{x_1, x_2, \dots, x_m\}$.

All solution procedures of linear programs require to be in some kind of table form before they may be applied. Different forms are often used for different types of problems. Procedures, too, may vary according to the type of problems. This amount of diversity can be not only confusing to the beginner but quite uneconomical and time-consuming in practice. It was with the hope of remedying this situation that the Symmetric Method was developed.

Every system of linear inequalities, virtually without any modification of original data and without addition of any arbitrary, unnecessary elements, may be represented by standard bilinear form, an abstract structure, equivalent to the given dual linear program. It contains all necessary information and is of ultimate condensed form, and at every stage of "realization" (solution) has the same size. It is called the "Standard Form":

0	\uparrow	\uparrow
\downarrow	A	b
\downarrow	c	0

In addition to compound activity matrix, it has the identification marginals of bilinear forms. Because the coordinate's notation is immaterial, the coordinates of optimization vector are identified by \uparrow and the coordinates of minimization vector by \downarrow . The first cell in the upper left corner serves for enumeration of iteration tables, the last right corner cell has the functional value. Once this form is given, it suffices for all linear problems, no matter of what type. A single, most general algorithm yields the solution without special modifications of data and in the smallest number of iterations.

The algorithm has two parts:

- The choice of the best permissible pivot, to have a fast solution without cycling. This choice is done for plus and minus pairs of values, symmetrically and simultaneously at every stage. The marginal-constraint values serve as indicators, even of consistency, degeneracy and termination.
- The elimination scheme. This scheme is universal elimination process of whole linear algebra, practically matrix inversion by complete partitioning. Because no accommodations are involved and the technique is strictly repetitive, this algorithm works inversely. In the case of Finite Matrix Game, we may get the original pay-off matrix from a given resolvent.

To illustrate how to set up problems and the Symmetric Method, we introduce three simple, typical linear programming examples:

II

A Deterministic Linear Programming.

Maximum Profit-Mixture Problem

Let us have a simple problem to maximize the objective function:

$$F(y) = 9y_1 + 7y_2 + 9y_3 + 5y_4 + 21y_5, \text{ subject to constraints:}$$

$$y_j \geq 0, j = 1, 2, \dots, 5 \text{ and}$$

$$y_1 + 3y_2 + 0y_3 + 2y_4 + 2y_5 \leq 7,$$

$$4y_1 + 3y_2 + 3y_3 + 3y_4 + 2y_5 \leq 16,$$

$$0y_1 + 4y_2 + y_3 + 4y_4 + 5y_5 \leq 17.$$

This problem has the corresponding dual minimum mixture problem:

$$\text{Minimize } F(x) = 7x_1 + 16x_2 + 17x_3, \text{ subject to constraints:}$$

$$x_l \geq 0, l = 1, 2, 3, \text{ and}$$

$$x_1 + 4x_2 + 0x_3 \geq 9,$$

$$3x_1 + 3x_2 + 4x_3 \geq 7,$$

$$0x_1 + 3x_2 + 1x_3 \geq 9,$$

$$2x_1 + 3x_2 + 4x_3 \geq 5,$$

$$2x_1 + 2x_2 + 5x_3 \geq 21.$$

Comparing these two problems, we observe that in both cases the numbers we have to find (x_l and y_j) are non-negative and that both systems involve ~~these~~ identical coefficients in two different systems of "weak" inequalities. It turns out, in mathematical theory, that one of these problems is solvable if, and only if, the other is. Moreover, in finding the solution of one, we always find the solution of the other automatically. It is obvious that there must exist one representation of such a dual problem in one form, that serves as a tool of solution. Our "Standard Form" is such a representation.

From now on, in the case of solution of linear programs and other linear problems, we will deal with such problems only in "Standard Form."

In the next table we have our first example:

	Standard Form					
$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{0}$
$\check{1}$	1	3	0	2	2	7
$\check{2}$	4	3	3	3	2	16
$\check{3}$	0	4	1	4	5	17
$\check{0}$	9	7	9	5	21	0

As already mentioned, every cell has its own meaning, from the first formulation of the problem, at every stage of "realization," to the last resolvent table. It has been found convenient, for desk calculations as well as for digital computer coding, to augment the basic compound activity matrix with marginal identifications, first row and first column.

1. The first cell (upper left corner), where we placed a zero in this table, serves for enumeration of tables in successive iterations (1, 2, ...).
2. The top marginal column (over the matrix of coefficients) cells have the coordinates of the maximizing vector $y = \{\hat{1}, \hat{2}, \dots, \hat{n}\}$ and the first left marginal column the coordinates of the minimizing vector $x = \{\hat{1}, \hat{2}, \dots, \hat{m}\}$. These marginals of the bilinear form help to keep track of the unknown for the purpose of solution (exchanges of coordinates, transformation-realization). The last cell in the marginal first row and column have \hat{o} and \hat{o} (o from optimum) serves to read the final result. Of course, the \hat{j} simply corresponds to y_j and \hat{i} to x_i . The notations of coordinates otherwise are immaterial for solution procedures.
3. The last column has the constraints b_i (in our case) and the last row the constraints c_j . They may be positive or negative in the given table and both serve as indicators, for selection of pivot (which may be positive or negative), for iteration, consistency, degeneracy, and termination at every stage of realization.
4. The last cell (lower right) has the value of the functional; in the formulation table, we place a zero there.

We are now ready to apply the solution algorithm of the Symmetric Method. It is a pivotal, direct, repetitive computational method of whole linear algebra.

The Symmetric algorithm for Linear Programs (including Two-Person Zero-sum Games) has two parts:

- a. Selection of pivot. This pivot-selection algorithm differs from any known. It is based on duality and skew-symmetry of bilinear forms which represent in the "Standard Form" systems of weak inequalities. The main objective of this selection algorithm is to eliminate cycling, to find the best permissible pivot, or the "nucleous matrix" as fast as possible and to solve, without any basic feasible solutions, both problems, primal and dual at once, symmetrically and simultaneously.
- b. Realization algorithm. This elimination algorithm has been found independently of any scheme and it is basically matrix inversion by complete partitioning. It is really a condensed (known since 1916) scheme of Burgess' elimination procedure

as shown by Petrie in 1951,⁵ but generalized for any pivot cell as a necessity for linear programs. The freedom of choice of any pivot is a most important fact for the direct possible solution of linear inequalities. It makes the form ultimately condensed, and because no accumulations are involved, it works inversely. In this respect, it is comparable only to the classical method of determinant and superior to every other elimination method, as far as we know. There are not involved any artificial elements and no duplicity in size of structure. Nevertheless, properties and information of the given system are preserved, even the slack vectors (if the inequalities would be changed on equations) are given in the resolvent. The extended algorithm, in the case of two-person zero-sum games, gives a characteristic kernel of the game, an analog to continuous games. This scheme, "realization algorithm," solves, of course, with complete freedom of pivot, any system of linear equations, matrix inversion and determinant evaluation, in particular.

Going back to our first special numerical example, we state the criteria which the pivot must satisfy for the first iteration. In this case, because all elements of matrix of coefficients and constraints are positive, for the first iteration:

- a. Find the smallest ratio

$$\frac{\hat{0}_i}{a_{ij}} \text{ for all } j \text{ columns (because all constraints } c_j \text{ are +).}$$

- b. Every column has at least one possible pivot.

Find the largest ratio

$$\frac{\sum_j \hat{0}_j \cdot \hat{0}_i}{a_{ij}}, \text{ which will correspond to the best permissible pivot.}$$

In our example, the largest ratio $\frac{(21)(17)}{5}$ gives us a pivot in the cell $(\hat{3} \hat{5})$, with $a_{35} = 5$. Once we have the selected, best admissible pivot, we may proceed to the first iteration, to the first linear transformation. It may be done this way:

1. Prepare a new complete blank form, of the identical pattern and size of table 0.
2. Add one to the number in the first cell (upper left-hand corner); in our case, write 1 there.
3. Exchange the symbols at the top of the pivot column with the symbol at the left of the pivot row. In our example, interchange the 3 and 5 symbols of coordinates.
4. Write in the pivot cell of the new table the reciprocal value of the original number, of $\frac{1}{a_{hk}}$, in our case $1/5$.

5. Divide every number of the pivot row, inclusive of the marginal cell under 0, by a_{hk} , in our case by 5.
6. The numbers in the pivot column are also divided by the negative pivot; (or divided by pivot and change signs), including the last (0) cell.
7. Every other cell element of table 1 is found by common elimination algorithm (including marginals and functional value in the cell 0 0).

Schematically written, for the given table and pivot p , column value c , row value r and any general element, we write:

$$e' = e - \frac{c \cdot r}{p} \quad \text{and for all four numbers involved}$$

0		
	e	c
	r	Ⓟ

→

1		
	e'	-c/p
	r/p	1/p

Applications of these rules to our table 0 give us table 1:

1	Ⓜ	Ⓐ	Ⓢ	Ⓐ	Ⓢ	Ⓟ
✓ 1	1	7/5	-2/5	2/5	-2/5	1/5
✓ 2	4	7/5	13/5	7/5	-2/5	46/5
Ⓐ 5	0	4/5	1/5	4/5	1/5	17/5
✓ 0	9	-49/5	24/5	-59/5	-21/5	-357/5

Application of the first rule leads to the solution. We continue now, selecting the next pivot. The permissible pivot will be positive (because there is no negative element in the last marginal column 0) and only in the column for which we have these pairs:

$$+ \quad (a_{ij})$$

$$+ \quad (\overset{\vee}{o}_j)$$

In our case, the possible pivot may be in the column (Ⓜ) or (Ⓢ) only. The smallest ratios in these two columns are for elements in cells (1 Ⓜ) and (2 Ⓢ). Because

$$\frac{9}{5} < \frac{(24) \cdot (46)}{(5) \cdot (13)}$$

the best pivot is in ($\hat{2}$ $\hat{3}$). [The more familiar Simplex method would have the pivot, in this case, in ($\hat{1}$ $\hat{1}$)]. By the same procedure as before, we get this table:

2	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$
$\hat{1}$	21/13	21/13	2/13	8/13	-6/13	21/13
$\hat{3}$	20/13	7/13	5/13	7/13	-2/13	46/13
$\hat{5}$	-4/13	9/13	-1/13	9/13	3/13	35/13
$\hat{0}$	21/13	-161/13	-24/13	-187/13	-45/13	-1149/13

This table is not yet a resolvent because the first column indicator in the cell ($\hat{0}$ $\hat{1}$) is positive. At least one iteration more will be a necessity. Because all the row indicators are once more positive, we will not have a pivot with a negative element in this example. The smallest ratio is for the cell ($\hat{1}$ $\hat{1}$), which is the pivot for the next and last iteration.

The elimination procedure for the new pivot is the same as before. It yields the table 3, which is the resolvent, because all row-indicators are positive and all column-indicators are negative.

3	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$
$\hat{1}$	13/21	1	2/21	8/21	-6/21	1
$\hat{3}$	-20/21	-1	5/21	-1/21	6/21	2
$\hat{5}$	4/21	1	-1/21	17/21	3/21	3
$\hat{0}$	-1	-14	-2	-15	-3	-90
$\hat{0} (-1)$	1	14	2	15	3	90

The last stage of the symmetric method we demonstrate in the added row. We multiply simply the marginal row ($\hat{0}$) by (-1) and read the result:

1. The optimal functional

$$F_0 = 90$$

2. The maximum vector

$$y = \{1, 0, 2, 0, 3\}.$$

The values of coordinates $\hat{1}$, $\hat{2}$, $\hat{3}$, $\hat{4}$, $\hat{5}$ are found as follows: the values of $\hat{1}$, $\hat{3}$ and $\hat{5}$ are read from the last column; the coordinates $\hat{2}$ and $\hat{4}$ did not exchange basis; they are 0. In the last marginal row, we find the corresponding slacks values for these coordinates $\hat{2}$ and $\hat{4}$; 14 and 15 respectively.

3. The minimum vector

$$x = \{1, 2, 3\}.$$

These two slacks just indicate what the difference in the C_j constraints would be if the inequalities were changed to equations.

For example, for $c_2 = 7 + 14$, we get

$$a_{12}x_1 + a_{22}x_2 + a_{32}x_3 \approx c_2, \text{ or}$$

the difference $21 - 7$ is the slack 14.

Similarly,

$$a_{14}x_1 + a_{24}x_2 + a_{34}x_3 \approx c_4, \text{ where } 20 = 5 + 15$$

The resolvent, in particular, contains the complete solution of bilinear form of $Ax = b$ and $A'y = c$, the inverse matrix, the adjoint matrix, the value of the determinant of non-zero coordinate system. This is, of course, a special case for a_{ij} positive and $x_j, y_i \approx 0$, which will not always happen.

The objective of this problem: The solution of a system of linear inequalities directly is a matrix manipulation without any alteration or addition, from one standard form and without the change in the system of equations. The Resolvent may be written in the following form, of columns and rows:

3'	1	2	3	4	5	0
1	13/21	2/21	-6/21	1	8/21	1
2	-20/21	5/21	6/21	-1	-1/21	2
3	4/21	-1/21	3/21	1	17/21	3
0	-1	-2	-3	-14	-15	-90

For the non-degenerate case, this pseudoinverse matrix is unique. In addition to A^{-1} , we get a submatrix

$$\begin{bmatrix} 1 & 8/21 \\ -1 & -1/21 \\ 1 & 17/21 \end{bmatrix}$$

The symmetric algorithm, from the resolvent will recover in three inverse iterations, the original activity matrix in standard form.

There does not exist any other direct method which, without a change of system inequalities to equations, would solve the problem of Linear Programming of matrix games as a simple matrix operation. The Symmetric Method, in the original size of the given table, proceeds step by step and in the same size gives the solution and all slack vectors.

III

Minimum Cost Linear Program The Duality and Formulation of the Problem

A dog needs a good meat diet, fortified with vitamins and protein. For example: A special breed of dog needs 12 units of protein per week, but not more than 10 units of fat. These values are so-called constraints.

Only three meats may be used to make up the diet: calves' liver, pork and veal. (Of course, this is an artificial, hypothetical example.)

Cost:	1 pound liver, \$.90;	contains 4 units protein
		1 unit fat
	1 pound pork, \$.40;	contains 2 units protein
		4 units fat
	1 pound veal \$.60;	contains 2 units protein
		2 units fat

The problem is to find the cheapest "Diet" which satisfies the dog's needs.

The problem looks more resolvable in the form of linear inequalities. Let the quantities of

liver	be	x_1 ,
pork	be	x_2 ,
veal	be	x_3 .

We shall have these two inequalities and one equation:

$$(1) \quad 4x_1 + 2x_2 + 2x_3 \geq 12 \quad (\text{protein, at least}).$$

$$(2) \quad x_1 + 4x_2 + 2x_3 \leq 10 \quad (\text{fat, no more}).$$

Minimize the value of cost (the objective function):

$$(3) \quad \text{Min } F(x_1, x_2, x_3) = .90x_1 + .40x_2 + .60x_3.$$

First, a necessary step must be taken. We simply change everything to a real minimum problem. It is simply done if (2), the inequation, is multiplied by (-1). We get

$$(2') \quad -x_1 - 4x_2 - 2x_3 \geq -10.$$

The problem is a proper minimum problem; we do not need to care about any feasible solution and a uniform + or - marginal as some methods need.

We have

$$(3) \quad 4x_1 + 2x_2 + 2x_3 \leq 12$$

$$-x_1 - 4x_2 - 2x_3 \leq -10$$

$$\text{Min } (.90x_1 + .40x_2 + .60x_3).$$

We may solve this problem directly by symmetric algorithm. But to keep simple things simple, it is much better to transpose this problem on its dual-maximum Linear Problem. We have here the profit function as a constraint and the constraint function as an objective function.

We may write (for a new vector $\{Y = y_1, y_2\}$)

$$(4) \quad 4y_1 - y_2 \leq .90$$

$$2y_1 - 4y_2 \leq .40$$

$$2y_1 - 2y_2 \leq .60$$

$$\text{Max } (12y_1 - 10y_2).$$

From this we get a real simple

Standard Form

0	\uparrow	\downarrow	$\hat{0}$
\checkmark 1	4	-1	.90
\checkmark 2	②	-4	.40
\checkmark 3	2	-2	.60
\checkmark 0	12	-10	0

Solution:

- (1) Only one column (\uparrow) in question, the first one. The smallest ratio $\frac{\hat{0}}{a_{.1}}$ is pivot. $\frac{.90}{4} = .225 > \frac{.40}{2} = .20 < \frac{.60}{2} = .30$.

Pivot is at ($\checkmark \uparrow$).

Prepare the blank table, and

1	↘ 2	↗ 2	↑
↘ 1	-2	(7)	.10
↑	1/2	-2	.20
↘ 3	-1	2	.20
↘ 0	-6	14	-2, 40

Write:

- (a) in first square upper left, 1, because first elimination.
- (b) exchange the notations of the pivot; this is what the first iteration just will perform. Leave the other coordinates unchanged.
- (c) replace the pivot with its reciprocal (1/2).
- (d) find the profit value (0 0): $0 - \frac{(12) \cdot (.40)}{2} = -2, 40$
- (e) divide all other elements of pivot row by 2.
- (f) divided all other elements of the pivot column by 2 and change the signs.
- (g) find remaining elements by elimination algorithm.

Now, the next elimination:

We have a (↑) problem once more. The smallest ratio is obviously $\frac{.10}{7}$, so the pivot is at (↘ 2) cell.

The next solution proceeds similarly:

2	↘ 2	↘ 1	↗ 2
↗ 2	-2/7	1/7	$\frac{1}{70}$
↗ 1	$\frac{1-4}{2 \quad 7}$	+2/7	$.20 + \frac{2}{70}$
↘ 3	$-1 + \frac{4}{7}$	-2/7	$.20 - \frac{2}{70}$
↘ 0	-2	-2	-2, 60

=

2'	↘ 2	↘ 1	↗ 2
↗ 2	-2/7	1/7	1/70
↗ 1	-1/14	2/7	8/35
↘ 3	-3/7	-2/7	6/35
↘ 0	-2	-2	-2, 60

We get in decimal fractions (multiply at the same time 0 row by (-1)):

3	\checkmark_2	\checkmark_1	$\hat{0}$
$\hat{2}$	-2/7	1/7	.014
$\hat{1}$	-1/14	2/7	.229
\checkmark_3	-3/7	-2/7	.171
\checkmark_0	2	2	2.60

Results and the Interpretation:

(1) The minimum cost of such a diet is \$2.60.

(2) It will consist of

$\checkmark_1 = x_1 = 2$ pounds of calves' liver, and

$\checkmark_2 = x_2 = 2$ pounds of pork,

$\checkmark_3 = x_3 = 0$ pounds

Any diet with veal would be more expensive; this is the unique minimum. We write it in vector:

$$X = \{2, 2, 0\}.$$

The dual problem does not mean fat or protein, but rather "shadow prices":

$$Y = \{8/35, 1/70\}.$$

(3) The optimum (minimum protein and maximum fat) is achieved at the same time.

(4) From a mathematical point of view this property is more important: The real "nucleus" of the structure is a sub-matrix

$$M = \begin{bmatrix} 4 & -1 \\ 2 & -4 \end{bmatrix}$$

(a) This M has a value $|M| = -14$

(b) The inverse matrix is

$$M \rightarrow \begin{bmatrix} 1/4 & -1/4 \\ -1/2 & -14/4 \end{bmatrix} \rightarrow \begin{bmatrix} 2/7 & -1/14 \\ 1/7 & -2/7 \end{bmatrix} = M^{-1}$$

This is what we have in the solution matrix:

$$\begin{array}{cc}
 \begin{array}{c} \nearrow \\ \nwarrow \end{array} & \begin{array}{c} \nwarrow \\ \nearrow \end{array} \\
 \begin{array}{c} \nwarrow \\ \nearrow \end{array} & \begin{array}{c} \nearrow \\ \nwarrow \end{array}
 \end{array}
 \begin{array}{|cc|}
 \hline
 -2/7 & 1/7 \\
 -1/4 & 2/7 \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{cc}
 \begin{array}{c} \nwarrow \\ \nearrow \end{array} & \begin{array}{c} \nearrow \\ \nwarrow \end{array} \\
 \begin{array}{c} \nwarrow \\ \nearrow \end{array} & \begin{array}{c} \nearrow \\ \nwarrow \end{array}
 \end{array}
 \begin{array}{|cc|}
 \hline
 2/7 & -1/4 \\
 1/7 & -2/7 \\
 \hline
 \end{array}
 = M^{-1}$$

IV

Typical Deterministic Linear Program

Let us have this mixture-optimization linear programming problem:

$$(1) \quad P_{\min}: \text{Min } (b, x) \text{ for } xA \geq c; \quad 0 \leq x \in V_m,$$

and/or

$$P_{\max}: \text{Max } (c, y) \text{ for } Ay \leq b; \quad 0 \leq y \in V_n,$$

given by these relations:

$$\begin{aligned}
 (2) \quad & 2x_1 - x_2 + 3x_3 \geq 2, \\
 & x_1 + 3x_2 - 2x_3 \leq 3, \\
 & x_1 + x_2 + x_3 \geq 2, \\
 & 3x_1 - 2x_2 + 2x_3 \leq 1,
 \end{aligned}$$

$$(3) \quad \text{Minimize } (-8x_1 + 2x_2 - x_3).$$

Multiplications of the second and fourth inequality in (2) by (-1) and a transposition of the given minimum problem to the maximum-dual problem will give:

$$\begin{aligned}
 (4) \quad & 2y_1 - y_2 + y_3 - 3y_4 \leq -8 \\
 & -y_1 - 3y_2 + y_3 + 2y_4 \leq 2 \\
 & 3y_1 + 2y_2 + y_3 - 2y_4 \leq -1
 \end{aligned}$$

$$(5) \quad \text{Maximize } (2y_1 - 3y_2 + 2y_3 - y_4).$$

The inequations (4) and (5) may be written now in the standard form and solved by the Symmetric Method (See next Table 0). Both constraints involve positive and negative values, and both positive and negative pivot-values are permitted. This example is solved in the next three tables without any cycling. Particular care is necessary only, in addition to the algorithm demonstrated in the last two problems, in deciding if a positive or negative pivot is the best permissible pivot. The

important difference between the Simplex method and the Symmetric method is obvious just from the solution of our third example. In the simple cases of the first and second problem, besides the condensation and different scheme of realization, the treatment of problems by both methods is very similar. But the solutions of our third example, if done and compared by both methods, differ remarkably.

In the starting table 0 of this third problem, both constraints (marginals) have the negative and positive values. It is clear here that in addition to differences already mentioned, the Symmetric method is an extension of the Simplex method, because it solves such mixed inequality systems, as in our table 0 directly, for negative pivot values and with negative values in the last constraint column, without any change in equations and without any feasible solution required before the beginning of the "realization" procedure. This is a very practical and economic feature. The conciseness, greater flexibility, broader scope in formulation of problems as well as the lack of redundancy with which the calculations take place are results of this feature. The striking simplicity and fruitfulness of the Symmetric method is more obvious at the solutions of the two-person zero-sum game problems, which will be discussed in later communications.

The pivots of the third example are found by this algorithm:

1. Find all possible pivots. For the column indicator (+), the pivot value is positive. For the row indicator (-) the pivot value will be negative.
2. The possible positive pivot in the column (+) is the pivot which corresponds to the lowest ratio (algebraically) of the ratios δ_i / a_{ij} . From all positive pivots the best has the highest (algebraically) value:

$$(\delta_j) \cdot (\delta_i) / a_{ij}$$

3. The possible negative value pivots for (--) rows we get similarly. The possible negative value pivot for each such row corresponds also to the lowest ratio (algebraically) of $(\delta_j) / a_{ij}$. If several rows are involved, the best negative pivot corresponds to the smallest value (algebraically) of

$$(\delta_j) \cdot (\delta_i) / a_{ij}$$

For the decision, if negative or positive pivot should be used, the rule here is this: If pivots of both signs are in question, find a value of the iteration

$$\frac{(\check{0}_j) (\hat{0}_j)}{p}$$

for both and add them algebraically. If the sum is positive, take the positive pivot; if negative, use the negative pivot.

The next four tables, with pivot cells marked, demonstrate the method and many advantages.

Standard Form:

$\check{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{0}$
$\check{1}$	2	-1	1	-3	-8
$\check{2}$	-1	-3	1	2	2
$\check{3}$	3	2	1	-2	-1
$\check{0}$	2	-3	2	-1	0

1	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\check{1}$	$\hat{0}$
$\hat{1}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{8}{3}$
$\check{2}$	$\frac{1}{3}$	$-\frac{11}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$-\frac{10}{3}$
$\check{3}$	$\frac{5}{3}$	$\frac{8}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{13}{3}$
$\check{0}$	$\frac{4}{3}$	$-\frac{8}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	$\frac{8}{3}$

2	$\hat{1}$	$\check{2}$	$\hat{3}$	$\check{1}$	$\hat{0}$
$\hat{1}$	$-\frac{7}{11}$	$\frac{1}{11}$	$-\frac{2}{11}$	$-\frac{3}{11}$	$\frac{26}{11}$
$\hat{2}$	$-\frac{1}{11}$	$-\frac{3}{11}$	$-\frac{5}{11}$	$-\frac{2}{11}$	$\frac{10}{11}$
$\check{3}$	$\frac{21}{11}$	$\frac{8}{11}$	$\frac{17}{11}$	$-\frac{2}{11}$	$\frac{21}{11}$
$\check{0}$	$\frac{12}{11}$	$-\frac{8}{11}$	$\frac{5}{11}$	$-\frac{9}{11}$	$\frac{56}{11}$

3	$\check{3}$	$\check{2}$	$\hat{3}$	$\check{1}$	$\hat{0}$
$\hat{1}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	3
$\hat{2}$	$\frac{1}{21}$	$-\frac{5}{21}$	$-\frac{8}{21}$	$-\frac{4}{21}$	1
$\hat{1}$	$\frac{11}{21}$	$\frac{8}{21}$	$\frac{17}{21}$	$-\frac{2}{21}$	1
$\check{0}$	$-\frac{4}{7}$	$-\frac{8}{7}$	$-\frac{3}{7}$	$-\frac{5}{7}$	4
$\check{0}(-1)$	$\frac{4}{7}$	$\frac{8}{7}$	$\frac{3}{7}$	$\frac{5}{7}$	-4

The results of this example are :

(1) optimum functional $F_0(x) = F_0(y) = -4$

(2) $y = \{1, 1, 0, 3\}$

(3) $x = \left\{ \frac{5}{7}, \frac{8}{7}, \frac{4}{7} \right\}$

V

The theory and mathematical background of the symmetric method may be found in notes (6) and (7) at the end of this article. More general discussions of the symmetric algorithm and applications to the solution of matrix games are prepared for publication. The symmetric method is programmed for Marquette University's International Business Machine 650 digital computer for both cases: for linear programs and for two-person zero-sum games in particular. The Standard Form for the two-person zero-sum game for the pay-off matrix $[a_{ij}]$ with non-negative elements (which it is always possible to have) is:

0	\uparrow	\uparrow
\downarrow	A	1
\downarrow	1	0

In this matrix, both constraint vectors have only 1's as elements; and we simply maximize the reciprocal value of the game, the value $1/V$ for player II (we minimize indirectly the value of the game V). The concept of algebraic structure (note 8) is also most promising for other aspects of the linear program; and the symmetric method, otherwise of striking simplicity, is very fruitful.

To give full credit to pioneers in the field, we have to mention Dantzig,³ who was the first to discover that a direct general solution of linear programs exists, and who developed the Simplex method for the solution of these problems. In the Simplex method, the slack vectors serve as identification factors of coordinates. The pivotal method, used in determinant evaluation, has been developed by Chio.² The pivotal elimination method, used in solutions of linear equations and in matrix inversion, was first used by Burgess¹ and was published in 1916. Dantzig's Simplex method is basically this Burgess elimination procedure, with the choice of pivot. There are many other elimination techniques used in linear algebra, and there are several other methods which solve

particular problems of linear programs. Petrie⁵ was the first who, in 1951, used a condensed scheme of Burgess', which corresponds to our scheme, for matrix inversion for pivots only on the main diagonal. The symmetric elimination scheme has free choice of pivot, because the identification of interchanges of coordinates is preserved by bilinear form.

Otherwise, some condensed simplex-tableaus for particular examples of linear programs with short-cut elimination schemes have been used sporadically in recent years elsewhere, always for systems of equations, by Tucker,⁹ Gomory,⁴ and Vajda.¹⁰

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